THEORETICAL POLARISATIONS OF HIGH FREQUENCY RADIO WAVES AT A LOW LATITUDE STATION

C. ABHIRAMA REDDY

IONOSPHERIC RESEARCH LABORATORIES, ANDHRA UNIVERSITY, WALTAIR.

(*Received, November* 3, 1962)

ABSTRACT. The polarisations of vertically propagated high frequency radio waves have been evaluated from the Appleton-Hartree formula for the latitude of Waltair (Geomagnetic Lat. 7.4°N, Dip angle 20°) by the rigorous computational method. The variations of the axial ratio and the tilt angle of the polarisation ellipse with electron density, collisional frequency and wave frequency are directly delineated so as to make possible a ready comparison with experimentally measured values of the above parameters.

INTRODUCTION

There are two principal methods of numerically evaluating the theoretical polarisations of ionospherically propagated radio waves from the Appleton-Hartree formula. One is the graphical method developed by Bailey (1934) and another is the computational method, which was also outlined by Bailey (1938). Both of these methods have been used by many workers for obtaining the general curves of polarisation parameters for a variety of propagation conditions (Martyn 1935, Murthy and Khastgir 1960; Ghosh 1938; Singh and Murthy 1958; Scott 1950; Taylor 1933; 1934; Snyder and Hellivell 1952 and Ratcliffe 1959).

It is the purpose of the present communication to describe the results obtained from an extensive series of calculations of polarisation parameters for vertical propagation conditions at Waltair, which is a low latitude station (Geomag. Lat. 7.4°N, Dip angle 20°).

DESCRIPTION OF THE METHOD

The magneto-ionic equation for the complex polarisation ($\bar{R}$) of the radio waves can be written in the form .

$$R = i[+(\eta+i\xi) \pm \{(\eta+i\xi)^2+1\}^{\frac{1}{2}}] \qquad \dots \quad (1)$$

where

$$\eta = X/(X^2+Y^2)\ ;\ \xi = Y/(X^2+Y^2)$$

$$X = \frac{(1-x)p}{\nu_c}\ ;\ Y = \nu/\nu_0$$

*Communicated by Prof B. Ramachandra Rao.

$$x = \frac{4\pi Ne^2}{mp^2} \cdot z = \nu/p$$

$$\nu_\theta = \left| \frac{P_H \sin^2 \theta}{2 \cos \theta} \right|$$

$$P_H = \frac{He}{mc} = \text{angular gyrofrequency of electrons}$$

H = Strength of the earth's magnetic field

θ_p = Angle between the direction of propagation and the direction of the magnetic field.

p = Angular frequency of the wave

ν = Collisional frequency of electrons

e, m = Charge and mass of an electron.

c = Velocity of e.m. waves in free space.

N = Number density of electrons.

Following the procedure outlined by Bailey (1938) eq. (1) can be transformed into the form :

$$R = r + is = i \left[(\eta + i\xi) \mp \left\{ \left(\frac{C+A}{2} \right)^{\frac{1}{2}} + i \left(\frac{C-A}{2} \right)^{\frac{1}{2}} \right\} \right] \qquad \ldots \quad (2)$$

where

$$A = \eta^2 - \xi^2 + 1.$$
$$B = 2\xi\eta$$
$$C = |(A^2+B^2)^{\frac{1}{2}}|$$

From a detailed analysis it can be shown that the upper negative sign in eq. (1) and (2) refers to the ordinary magneto-ionic component while the lower positive sign refers to the extraordinary component. Thus, denoting the ordinary wave polarisation by R_0, we can write .

$$R_0 = r_0 + is_0 \qquad \ldots \quad (3)$$

where

$$\left.\begin{aligned} r_0 &= -\left(\xi - \sqrt{\frac{C-A}{2}}\right) \\ s_0 &= \left(\eta - \sqrt{\frac{C+A}{2}}\right) \end{aligned}\right\} \qquad \ldots \quad (4)$$

Using the positive sign before the radical in eq. (2) and proceeding as above we obtain for the extraordinary wave

$$R_x = r_x + is_x$$

where

$$\left.\begin{aligned} r_x &= -\left(\xi + \sqrt{\frac{C-A}{2}}\right) \\ s_x &= \left(\eta + \sqrt{\frac{C+A}{2}}\right) \end{aligned}\right\} \quad \ldots (5)$$

Thus, strating from a pair of values of ξ and η, the polarisation parameters, r and s, of either characteristic wave can be calculated through A, B, C and equations (4) and (5). From the known values of r and s, the tilt angle (ψ) and the axial ratio (ϵ) of the polarisation ellipse can be calculated from the following set of eq. tions :

$$r^2 + s^2 = \rho^2 \tag{6}$$

$$\tan^{-1}(s/r) = \phi \tag{7}$$

$$\tan 2\psi = \frac{2\rho\cos\phi}{\rho^2 - 1} \tag{8}$$

$$\epsilon^2 = \tan^2\theta = \frac{\rho^2+1+\sqrt{(\rho^2-1)^2+4\rho^2\cos^2\phi}}{\rho^2+1-\sqrt{(\rho^2-1)^2+4\rho^2\cos^2\phi}} \tag{9}$$

$$\epsilon = \frac{\text{semi-minor axis}}{\text{semi-major axis}}$$

ψ = the angle between the major axis of the elipse and the magnetic north direction.

The electric vector is considered in all the above equations.

But, the above method of obtaining ψ and ϵ corresponding to a set of r and s values is rather laborious, and a simpler graphical method, which gives the parameters ψ and ϵ to an accuracy sufficient for comparison with experimental values, is used. This method is based on the analysis given by Booker (1934) of the complex polarisation R which is represented in a complex plane. He had shown that all possible polarisations can be known from points in the first quadrant of a unit circle in the complex R-plane, by using proper signs and inversions depending upon the signs of X and Y. Therefore, if a chart is prepared containing lines of constant ψ and constant θ in this first quadrant, then the values of ψ and θ corresponding to any particular set of r and s values can be read directly from this chart. The following two equations, relating r and s to the parameters ψ and θ, can be derived from equations (8) and (9).

$$\left(r-\frac{1}{\tan 2\psi}\right)^2+s^2 = 1/\sin^2(2\psi) \quad .. \quad (10)$$

$$\left(s\pm\frac{1}{\sin 2\theta}\right)^2+r^2 = 1/\tan^2(2\theta) \quad ... \quad (11)$$

Each of the above is an equation for a circle Thus, the curves of constant ψ are circles with centres at $(1/\tan 2\psi, 0)$ and with radii, $(1/\sin 2\psi)$, the curves of constant θ are circles with centres at $(0, \pm 1/\sin 2\theta)$ and radii $(1/\tan 2\theta)$. A chart of 20 cm $\times$ 20 cm size with 1 mm divisions has been used for obtaining ψ and θ by the above method.

Each of the parameters ξ and η involves four variables namely, the electronic density, the collisional frequency, the wave frequency and the critical collisional frequency. For vertical propagation, the critical collissional frequency ν_c, which is determined by the strength and inclination of earth's magnetic field with respect to the direction of propagation, is uniquely fixed at any one place if its height variation in the ionosphere is neglected. Therefore, the value of X is determined by x and p and that of Y by ν. By assigning the desired values to p, x and ν, sets of values of X and Y can be tabulated and the corresponding values of r and s evaluated

Computations have been made starting with discrete values of X and Y the possible ranges of which have been determined from the chosen ranges of wave frequency and the collisional frequency. The chosen wave frequency range is from 1 to 6 Mc/s The critical collisional frequency for the latitude of Waltair being $8.63 Mc/s$, the value of X varies from zero to a maximum of 4.37 in the above wave frequency range if propagation within the x-range of $0-1.0$ is considered. The collision frequency (ν) of electrons varies from about $5\times10^6 C/s$ at the lower fringe of the E-region to about $10^3 C/s$ in the region of maximum ionization in the F-layer (Ratcliffe 1960, Nicolet 1959) For the wave frequencies chosen, this range of collisional frequency values is relevant under different possible conditions of ionospheric propagation Therefore, ten values of $Y(=2\nu/\nu_c)$ in the range of 0 to 2.0 have been used in the computations.

It may be mentioned that the quantities X and Y above are the same as the X and Z, respectively, in Snyder and Helliwell's graphs (Snyder and Helliwel 1952), and the $(-\xi)$ and $[\nu/|\omega_c|]$ in Ratcliffe's curves (Ratcliffe, 1959, p. 72).

RESULTS AND DISCUSSION

The results of theoretical calculations for the ordinary wave are presented graphically in figures 1 to 4. It is usual to represent the variation of polarisation with electron density by means of $\theta-N$ and $\psi-N$ curves. Such representation has the advantage that both the sets of curves can be shown in the same graph in a compact from. But, with a view to facilitate a direct comparison of the theoretical and experimental values of polarisation parameters that are actually measured, curves have been drawn showing the variation of the axial ratio (ϵ or $\tan\theta$) and the tilt angle (ψ) with the electronic density for different fixed values of collisional frequency. Fig. 1 shows the variation of ϵ with x for a fixed wave

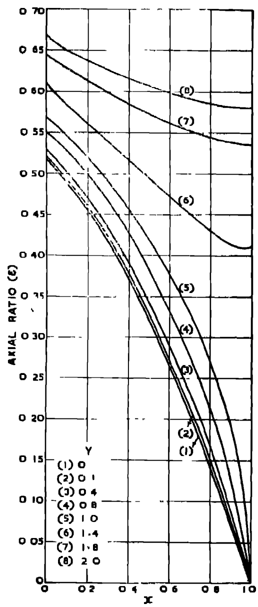

Fig 1 Variation of axial ratio with electron density.

frequency of $2Mc/s$ and for eight different values of Y in the range of 0 to 2. The important conclusions that can be drawn from there curves are as follows ·

(i) There is a marked variation with electronic density in the ellipticity for all values of collisional frequency.

(ii) The rate of variation of ϵ with x over the x-range of 0 to 1 increases slowly at first with collisional frequency upto $\nu = \nu_c$, but for values of $\nu > \nu_c$, there is a sharp reduction in the variation of ϵ with x; the variation becomes very small for $\nu >> \nu_c$.

(iii) The effect of increased collisional frequency at any value of x is to increase the axial ratio, the effect being more marked at larger values of x.

(iv) The most important feature of practical interest is the negligibly small change in c, at any value of x, as Y changes from 0 to 0.1 corresponding to a collisional frequency change of zero to 8.63×10^5 C/s. The effect of collisions on the axial ratio becomes appreciable only when Y is about 0.5 or larger.

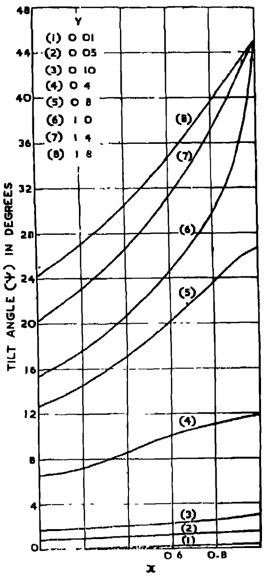

Fig. 2 Variation of tilt angle with electron density

The variation of the tilt angle (ψ) with x is shown in Fig. 2. The important features of this variation are as follows:

(i) The variation of ψ with x has a maximum range for $\nu = \nu_c$ and gets smaller as ν decrease below or increases above the value of ν_c.

(ii) For values of Y smaller than about 0.01, the variation of ψ with x is negligibly small.

(iii) The effect of collisional frequency on the tilt angle is much larger than the effect on axial ratios; thus, as Y changes from 0 to 0.1, ψ changes from 0 to 2°, which is a measurable change, while the corresponding change in ϵ is seen to be negligible.

(iv) The value of ψ changes appreciably with the collisional frequency for values of $Y > 0.10$.

From the above features, it may be concluded that the axial ratio is, in general, more sensitive to changes in electron density while the tilt angle is more sensitive to changes in collisional frequency.

One important feature observable in both the Figs. 1 and 2 is the marked difference in the shape of the curves as Y increases through unity. The axial ratio becomes zero at $x = 1$ for all values of $\nu < \nu_c$, while it has different values at $x = 1$ for different values of $\nu > \nu_c$. On the other hand, the tilt angle has different values at $x = 1$ for different values of $\nu < \nu_c$, while it shows the same maximum value of 45° for all values of $\nu > \nu_c$. It is to be noted, however, that this difference in the behaviour of the polarisation characteristics for $\nu > \nu_c$ and $\nu < \nu_c$ is absent for lower values of x, notably at $x = 0$, and the variations of ϵ and ψ are gradual.

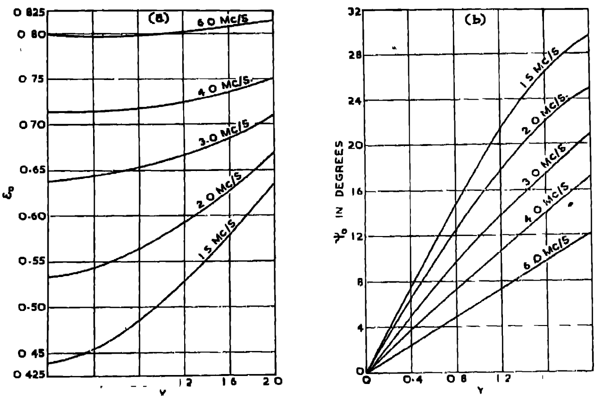

Fig 3 Variation of ε_0 and ψ_0 with collision frequency

The variations of ϵ and ψ with electron density are shown for a single frequency of 2 Mc/s; these curves are typical for the high frequency range and the general conclusions given above hold good for frequencies higher than 2Mc/s. The theoretical values of particular interest are those of the axial ratio and tilt angle at $x = 0$. These are the values that are experimentally measured when the radio waves, emerging out of the ionosphere, reach the ground. Some doubt has been expressed regarding the validity of the assumption that the final polarisation of the radio waves leaving the ionosphere corresponds to the level where $x = 0$ (Roy and Verma 1955). A detailed quantitative study of the limiting polarisation of radio waves has shown clearly that, in case of high frequency radio waves coming down vertically, the final limiting polarisation is acquired at a level in the ionosphere where the value of x does not differ significantly from zero. Therefore, these limiting values, ϵ_0 and ψ_0, of the axial ratio and tilt angle, respectively, at $x = 0$ are shown graphically in Fig. 3 and 4. Fig. 3(a) shows the variation of ϵ_0 with collisional frequency for five fixed wave frequencies. The following points of practical interest emerge from a study of these curves.

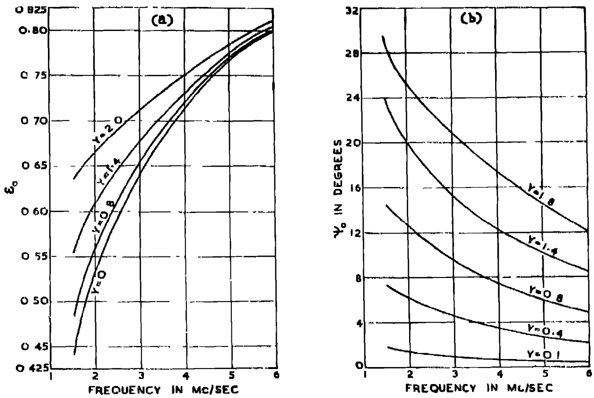

Fig 4 Variation of ϵ_0 and ψ_0 with wave frequency.

(i) At frequencies of 2Mc/s and higher, the axial ratio of the limiting polarisation ellipse for all values of $\nu \leqslant 0.5\nu_c$ differs by less than 1 per cent from the zero collisional frequency value.

(ii) The effect of collisional frequency on ϵ_0 diminishes rapidly as the wave frequency increases above 2Mc/s, the increase in ϵ_0 even at $\nu = 2\nu_c$ over that at $\nu = 0$ being less than one per cent at the wave frequency of 6Mc/s

The importance of the above theoretical facts becomes obvious when it is remembered that the collisional frequencies in the 'limiting region' where the down-coming waves in the frequency range of 2Mc/s and above acquire their final polarisation are not likely to be greater than $0.5\nu_c$ under normal ionospheric conditions. As such, the observed axial ratios in this frequency range should practically be the same as predicted by the magneto-ionic theory for the case of no collisions.

Fig 3(*b*) shows the variations of ψ_0 with Y for the same five fixed frequencies as in (a) The effect of collisional frequency is significant in this case, especially at frequencies 3Mc/s and below A collisional frequency of $0.1\nu_c$, which is likely to obtain in the 'limiting region' when it is situated at the lower fringe of the (mid-day) *E*-region, causes a radio wave of 2Mc/s to have a tilt angle of nearly 1.5° which is at the lower limit of measurable values of tilt angle. For lower values of ν, the tilt angle is too small to be measured accurately.

The variations of ϵ_0 and ψ_0 with the wave frequency are shown in Fig. 4, for fixed values of Y, to bring out clearly the dependence of the limiting polarisation on the wave frequency. The important points of interest to be noted from the curves are as follows.

(i) There is a marked variation of the axial ratio with frequency in the range of 1.5 to 6.0 Mc/s, the rate of variation being larger at lower wave frequencies and at smaller collisional frequencies.

(ii) The variation in tilt angle with wave frequency gets smaller for smaller values of collisional frequency and the variation is negligibly small in the practical range of ($Y \leqslant 0.10$) collisional frequency values when the wave frequency varies from 2 to 6 Mc/s.

(iii) The larger the wave frequency, the smaller is the effect of collisional frequency on the tilt angle of the limiting polarisation ellipse.

All the above results and discussions refer to the ordinary magneto-ionic component. It is well known that the ordinary and extraordinary wave polarisations are related to each other in a well defined way at each level of the ionosphere. As such the extraordinary wave polarisation is not discussed separately.

CONCLUSIONS

The detailed investigation of theoretical polarisations of radio waves at this low latitude station (Waltair) has shown that the experimental measurements on polarisation at low latitude stations in the frequency range of 1 to 6 Mc/s, should show a clear increase in the measured axial ratios with frequency and that the measured tilt angles should be rather too small to observe their variations with frequency. The results of the above study are in general agreement with those of the many previous workers referred to in the Introduction, with the exception of Mary Taylor's results. The conclusion of Mary Taylor (1933, 1934) that the limiting polarisation is always circular is found to be at variance with the results of all subsequent workers on polarisation and it lacks a theoretical basis.

ACKNOWLEDGMENTS

The author is indebted to Prof B. Ramachandra Rao, D.Sc., M. Brit. I.R.E., Sr. M.I.R.E., F.P.S. (Lond), for his keen interest and valuable guidance throughout this investigation. Thanks are also due to the Ministry of Education, Government of India, for the award of a Senior Research Scholarship to the author during the period of the above investigation.

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